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Asymptotic freedom and confinement from Type 0 string theory

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Abstract: We argue that there are generic solutions to the type 0 gravity equations of motion that are confining in the infrared and have log scaling in the ultraviolet. The background curvature generically diverges in the *IR*. Nevertheless, there exist solutions where higher order string corrections appear to be exponentially suppressed in the *IR* with respect to the leading type 0 gravity terms. For these solutions the tachyon flows to a fixed value. We show that the generic solutions lead to a long range linear quark potential, magnetic screening and a discrete glueball spectrum. We also estimate some WKB glueball mass ratios and compare them to ratios found using finite temperature models and lattice computations.

Keywords: Confinement, Brane Dynamics in Gauge Theories, Supergravity Models.

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Contents

1. Introduction

One of the many interesting developments to arise out of Maldacena's conjecture [1, 2, 3] concerns the study of large N QCD. While it is not known how to construct the dual gravity theory for SU(3) gauge theory with six quark flavors, nor even the dual for any pure SU(N) theory, it is hoped that the gravity duals that are known will share many of the properties of everyday QCD. The main properties we seek are asymptotic freedom in the ultraviolet and confinement in the infrared.

In four space-time dimensions, the Maldacena conjecture was originally applied to $\mathcal{N} = 4$ supersymmetric Yang-Mills [1] and then to other theories that have less supersymmetry but are still conformal [4]–[15]. Since the β-function is zero for these theories, they do not behave like ordinary QCD.

Witten has proposed supersymmetric Yang-Mills at finite temperature as a model more suited for comparison to QCD [16]. A d dimensional euclidean gauge theory at finite temperature is equivalent to a theory with $d-1$ noncompact directions and a Euclidean time compactified on a circle. At large distances the theory acts like a nonsupersymmetric euclidean Yang-Mills theory in $d-1$ dimensions. Hence, one expects to find area law behavior and a gap. The gravity dual for finite temperature super Yang-Mills is a nonextremal black hole in AdS space. Using this, Witten was able to argue that the bulk dilaton wave equation has a discrete spectrum, implying

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a gap for the boundary theory. He also demonstrated that there is an area law for the space-like Wilson lines. It was later shown that the finite temperature theory exhibits magnetic screening [17, 18]. There has also been a mini-industry comparing dual gravity results to lattice results [19], with some reasonable agreement [20, 21, 22, 23, 24, 25, 26, 27, 28].

However, in the UV the finite temperature theories effectively become supersymmetric with one extra dimension. Thus, the coupling does not run, but remains a free parameter. Related to this problem is that the QCD scale is set by the temperature instead of by dimensional transmutation. So the glueball mass scale is roughly the temperature T, while the string tension, with the free parameter, is g^2NT^2. In a QCD theory where the scale is set by dimensional transmutation, one expects the glueball mass scale to be directly related to the string scale.

We thus seek another gravity dual to better describe QCD. One such candidate is the type 0 model proposed by Klebanov and Tseytlin [29]. Ironically, this model was considered a toy model when first developed [30, 31]. Following an original suggestion by Polyakov [32], Klebanov and Tseytlin argued that one could construct the dual of an SU(N) gauge theory with 6 real adjoint scalars by stacking N electric D3 branes of the type 0 model on top of each other. The type 0 model has a closed string tachyon that lives in the bulk, but no open string tachyon that lives on the branes. The closed string tachyon couples to the five form field strength, which then drives the tachyon to a nonzero expectation value.

In [26] it was shown using the equations of motion derived in [29] that the effective coupling between two heavy quarks has the desired logarithmic fall off in the UV

$$g_{YM}{}^2 \sim \frac{1}{\log\mu}, \tag{1.1}$$

where μ is the energy scale. A possible IR solution was given in [26], but after a closer inspection, one finds that for generic tachyon potentials, the UV solution does not connect to this particular IR solution. In [33] a different IR solution was given which corresponds to an IR fixed point with infinite coupling and no confinement. For a given class of tachyon potentials, such a fixed point could conceivably connect to the UV point in [26].

At first glance, the presence of this IR fixed point seems sensible since, as was shown in [33], the two loop contribution to the β-function is positive. Hence one might expect a Banks-Zaks fixed point [34]. Presumably, quantum corrections can push the value of this fixed point to infinite coupling.

On the other hand, one could argue that quantum corrections lift the masses of the adjoint scalars. Hence this theory should behave much like pure SU(N) gauge theory, with some extra massive states. Using this reasoning, one should expect the type 0 theory to confine.

In fact, using only some modest assumptions about the tachyon potentials, we will show that the classical dual gravity equations of motion have generic solutions in the *IR* which exhibit *i*) a linear quark-antiquark potential *ii*) magnetic screening and *iii*) a discrete glueball spectrum. The glueball mass scale is related to the string scale by $m \sim \sqrt{\sigma}$, with no dependence on a free parameter.

If one assumes that the tachyon reaches a fixed value in the *IR*, then there exists *IR* solutions where the curvature diverges, but where many, if not all, α' corrections fall off exponentially.

In section 2 we review the type 0 model and discuss a *UV* solution with a running coupling constant. We assume here that the tachyon potentials satisfy a minimum number of properties, but are otherwise generic. In section 3 we discuss various *IR* solutions. We argue that in order to reach the *IR* solutions discussed previously in the literature requires plenty of fine tuning, and even then, it is not guaranteed that the *UV* solution of section 2 can match to these solutions. We then discuss a generic class of *IR* solutions and argue that the *UV* solution is adjustable enough to match over the range of these *IR* solutions. In section 4 we examine the validity of these solutions beyond the classical limit, where in general we find that string corrections swamp the leading order term. However, we find some solutions where many α' corrections are exponentially suppressed. In the *IR* these solutions approach a conformal transformation of the product space $R_+ \times M_4 \times S_5$, with the tachyon flowing to a constant value. This leads us to speculate that there is an exact solution to the full σ-model whose asymptotic behavior matches the asymptotic behavior of these classical solutions. In section 5 we discuss the physical implications of the generic *IR* solutions in section 3. Given one inequality, which is satisfied by the special solutions found in section 4, we argue that two heavy quarks have a linear potential at large distances. We further argue that there is magnetic screening and a discrete spectrum. We also calculate some glueball mass ratios using the WKB method. The results differ somewhat from the finite temperature results and the errors are larger due to the complexity of the equations. For the low lying masses we can make reasonable estimates for lower bounds of mass ratios which are consistent with lattice computations. In section 6 we present our conclusions.

2. The Type 0 nonsupersymmetric model

The type 0 model [30, 31] has a closed string tachyon, no fermions and a doubled set of R-R fields [30, 31], and thus a doubled set of D branes [35]. Because of this doubling of the R-R fields, one can relax the self dual constraint on the 5-form field strength. Hence one can have D3 branes that are electric instead of dyonic. The low energy world-volume action for N parallel electric D3 branes is SU(N) QCD with six real adjoint scalar fields, but no fermions. Hence, there is no supersymmetry and the coupling will run. There is no open string tachyon [35], so there is no tachyon in this QCD model.

Klebanov and Tseytlin argued that the massed squared of the *closed* string tachyon gets a positive shift from the background 5 form flux [29]. With a background flux the tachyon potential is not symmetric under $T \to -T$, so a large flux can drive the tachyon expectation value to a nonzero value that is basically independent of the tachyon bare mass term and its quartic coupling. The tachyon field is a source for the dilaton, thus unlike the $\mathcal{N} = 4$ case, the dilaton expectation is not constant.

One then makes the following ansatz for the metric [29]

$$ds^2 = e^{\frac{1}{2}\phi}\left(e^{\frac{1}{2}\xi - 5\eta} d\rho^2 + e^{-\frac{1}{2}\xi} dx_{||}^2 + e^{\frac{1}{2}\xi - \eta} d\Omega_5^2\right), \tag{2.1}$$

where ϕ, ξ and η are functions of ρ only. The equations of motion then reduce to a Toda like system with an action [29]

$$\begin{aligned} S &= \int d\rho \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\xi}^2 + \frac{1}{4}\dot{T}^2 - 5\dot{\eta}^2 - V(\phi,\xi,\eta,T)\right] \\ V(\phi,\xi,\eta,T) &= g(T) e^{\frac{1}{2}\phi + \frac{1}{2}\xi - 5\eta} + 20 e^{-4\eta} - Q^2 f^{-1}(T) e^{-2\xi}, \end{aligned} \tag{2.2}$$

and a constraint

$$\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\xi}^2 + \frac{1}{4}\dot{T}^2 - 5\dot{\eta}^2 + V(\phi,\xi,\eta) = 0\,. \tag{2.3}$$

That the left hand side of (2.3) is a constant follows from the equations of motion. The fact that the constant is zero follows directly from the dilaton equations of motion and the Einstein equation. Q is the total D3 brane charge which is proportional to N, T is the tachyon field and $f(T)$ is a function given by [29]

$$f(T) = 1 + T + \frac{1}{2}T^2\,. \tag{2.4}$$

This describes the tachyon coupling to the five form flux. The bare tachyon potential is the negative of $g(T)$

$$g(T) = \frac{1}{2}T^2 - \lambda T^4\,, \tag{2.5}$$

where we have included the quartic piece of the potential. The coefficient λ of the quartic coupling has yet to be determined unambiguously, so for the purposes of this paper, we will treat it as a parameter. Actually, the details of $f(T)$ and $g(T)$ are not that important, except that we require that $f(T)$ have a minimum at some finite value of T and that $g(T)$ is positive at that value. We will often consider the special case where $g'(T) = 0$ when $f'(T) = 0$.

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The equations of motion derived from the action in (2.2) are

$$\begin{aligned}
\ddot{\xi} + \frac{1}{2}g(T)e^{\frac{1}{2}\phi+\frac{1}{2}\xi-5\eta} + 2\frac{Q^2}{f(T)}e^{-2\xi} &= 0 \\
\ddot{\eta} + \frac{1}{2}g(T)e^{\frac{1}{2}\phi+\frac{1}{2}\xi-5\eta} + 8e^{-4\eta} &= 0 \\
\ddot{\phi} + \frac{1}{2}g(T)e^{\frac{1}{2}\phi+\frac{1}{2}\xi-5\eta} &= 0 \\
\ddot{T} + 2g'(T)e^{\frac{1}{2}\phi+\frac{1}{2}\xi-5\eta} + 2\frac{Q^2 f'(T)}{f^2(T)}e^{-2\xi} &= 0\,.
\end{aligned} \tag{2.6}$$

For large Q, $g(T)$ plays a secondary role to $f(T)$ and so the tachyon expectation value is determined by setting $f'(T) = 0$. As a first approximation, we may assume that the tachyon is a constant $T = T_0$. If $T_0 = 0$ then the solution reduces to the $\mathcal{N} = 4$ solution. When T is nonzero, then all fields are coupled and there is no known analytic solution. However, we can attempt to find approximate solutions that are valid in the UV and IR regions.

An asymptotic solution valid in the UV was given in [26]. As a first approximation one can take the dilaton field to be relatively constant, at least compared with ξ and η. Assuming a constant ϕ, the equations for ξ and η can be solved exactly, at least in the near horizon limit. In this case we find

$$e^{\xi} = C_1\rho \qquad\qquad e^{\eta} = C_2\rho^{1/2} \tag{2.7}$$

and

$$\begin{aligned}
\frac{1}{2}g(T_0)\frac{C_1^{1/2}}{C_2^5}e^{\frac{1}{2}\phi} + \frac{2Q^2}{C_1^2 f(T_0)} - 1 &= 0 \\
5g(T_0)\frac{C_1^{1/2}}{C_2^5}e^{\frac{1}{2}\phi} + \frac{80}{C_2^4} - 5 &= 0\,.
\end{aligned} \tag{2.8}$$

One can easily check that this satisfies the constraint equation in (2.3). If we plug this back into the metric, we find that the solution is still $AdS_5 \times S_5$, but the curvatures of the two spaces no longer match; S_5 now has smaller curvature then AdS_5. In this case the Ricci scalar for the total space is proportional to

$$R \;\sim\; g(T_0)e^{\frac{1}{2}\phi}\,. \tag{2.9}$$

Using the ξ and η solutions as inputs, we can go back and find an approximate solution for ϕ in terms of ρ. Using the ansatz $e^{\frac{1}{2}\phi} = C_0(\log(\rho/\rho_0))^{\alpha}$, and plugging this into the equation of motion for ϕ in (2.6), we find that the ansatz is a leading order solution if $\alpha = -1$ and $C_0 = -4C_2^5/(g(T_0)\sqrt{C_1})$. ρ_0 is an integration constant and we assume that $\rho_0 \gg 1$ in order that the gauge theory length scale is much greater than the string scale. Setting $\rho = u^{-4}$, and using the lowest order solutions for C_1 and C_2 from (2.8), we learn that the leading order behavior for the coupling is

$$e^{-\phi} = \frac{1}{g_{YM}^2} = \frac{Q g^2(T_0)\left(\log\frac{u}{u_0}\right)^2}{1024\sqrt{2f(T_0)}}\,. \tag{2.10}$$

One can easily check that to leading order in $1/\log u$, the constraint equation is still satisfied. We can also estimate the range of validity for this solution. Computing the leading order corrections to C_1 and C_2, one finds that

$$C_1 = \frac{2Q}{\sqrt{2f(T_0)}}\left(1+\frac{1}{4\log\frac{u}{u_0}}\right), \qquad C_2 = 2\left(1+\frac{1}{4\log\frac{u}{u_0}}\right). \tag{2.11}$$

We can also compare the terms in the potential that depend on the tachyon. Since

$$\frac{1}{2}g(T_0)e^{\frac{1}{2}\phi+\frac{1}{2}\xi-5\eta} \sim \frac{u^8}{2\log\frac{u}{u_0}}\,, \qquad Q^2 f^{-1}(T_0)e^{-2\xi} \sim \frac{u^8}{2}\,, \tag{2.12}$$

our solution with constant $T = T_0$ and $f'(T_0) = 0$ is valid so long as $\log(u/u_0) \gg 1$. While we are treating λ as a parameter, we should note that it is crucial that $g(T) > 0$ when $f'(T) = 0$. Hence we assume that $0 < \lambda < 1/2$. The lower bound on λ is so that the complete tachyon potential is bounded below.

The metric in the large u limit is

$$ds^2 = \frac{16}{g(T_0)\log\frac{u}{u_0}}\left(\frac{du^2}{u^2} + \frac{\sqrt{2f(T_0)}}{2Q}\left(1+\frac{1}{\log\frac{u}{u_0}}\right)u^2 dx_{||}^2 + \left(1+\frac{1}{\log\frac{u}{u_0}}\right)d\Omega_5^2\right). \tag{2.13}$$

Hence we can trust the classical dual gravity solution only if $g(T_0) \ll 1$, since $\log(u/u_0) \gg 1$. However, for generic values of λ, $g(T_0) \sim 1$ when $f'(T_0) = 0$, hence it would seem that the classical result is not particularly trustworthy. As it happens, the situation is not completely hopeless [33]. While the curvature for (2.13) diverges in the large u limit, the Weyl tensor actually falls off as

$$|C| \sim \frac{1}{\log u}\,. \tag{2.14}$$

This is a consequence of the conformal invariance in the large u limit, where the space-time approaches that of $AdS_5 \times S_5$, with divergent curvature. Because of this fall off in $|C|$, the α'^3 correction is of the same order as the classical term. Hence, while quantitative results might not be exact, the classical dual gravity solutions could capture the true qualitative behavior for this SU(N) gauge theory.

It would seem that the dual gravity calculation failed its first test: the prediction in (2.10) is that $e^{-\phi}$ has a log squared dependence instead of the linear log behavior found in perturbative Yang-Mills. However, the physical coupling is determined by

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finding the potential between two heavy quarks. Using the Wilson line computation of [36, 37], one finds that the quark potential is given by [26]

$$V \approx -\frac{128\,\pi^3}{\Gamma(\frac{1}{4})^4 g(T_0) L \log(\frac{L_0}{L})} \qquad L \ll L_0\,, \tag{2.15}$$

where L_0 is some length that can be adjusted to be much longer than the string scale. Hence, the effective coupling between a heavy quark and its antiquark *does* appear with the expected log dependence. In deriving (2.15) we have substituted $R^2 = \frac{16}{g(T_0)\log(u/u_0)}$ into the quark potentials found in in [36, 37]. A perturbative Yang-Mills computation gives a quark potential proportional to g^2N. Since the perturbative Yang-Mills coupling behaves like $g^2N \sim 1/\log(L_0/L)$, we find that the dual gravity computation results in an effective coupling with the desired log dependence. The actual coefficient is related to the one loop β-function. Unfortunately, it is clear that the coefficient is model dependent and we don't know enough to actually compute this using the gravity dual. Even if we did, we should not necessarily expect to get the correct answer since stringy corrections will be of order 1.

As we move away from the UV point at $u = \infty$, the tachyon will begin moving away from T_0. Solving the equations of motion to the next leading order we find that [33]

$$\begin{aligned}
T &= T_0 - 4\frac{g'(T_0)}{g(T_0)}\frac{1}{\log\rho} + \mathrm{O}\left(\frac{\log(-\log\rho)}{\log^2\rho}\right)\\
\phi &= -2\log(C_0\log\rho) - \left(7 + 8\left(\frac{g'(T_0)}{g(T_0)}\right)^2\right)\left(\frac{\log(-\log\rho)}{\log\rho}\right) + \frac{B}{\log\rho} + \\
&\quad +\mathrm{O}\left(\frac{\log^2(-\log\rho)}{\log^2\rho}\right)\\
\xi &= \log\left(\sqrt{2f^{-1}(T_0)}\;Q\rho\right) - \frac{1}{\log\rho} + \mathrm{O}\left(\frac{\log(-\log\rho)}{\log^2\rho}\right)\\
\eta &= \frac{1}{2}\log(4\rho) - \frac{1}{\log\rho} + \mathrm{O}\left(\frac{\log(-\log\rho)}{\log^2\rho}\right).
\end{aligned} \tag{2.16}$$

The constant B in the ϕ expansion is an integration constant that can be removed under a conformal rescaling. Notice that Q only appears as an overall factor in front of e^{ξ}. We also notice that the sign of $g'(T_0)$ determines whether or not the tachyon expectation value is driven toward or away from zero as one moves away from $\rho = 0$. There is also the possibility that $g'(T_0) = 0$, in which case the tachyon expectation value is unchanged as the system varies.

Another interesting point concerns the ratio of the two and one loop contributions to the β-function [33]. To next to leading order the effective coupling $e^{\frac{1}{2}\phi}$ is given by

$$e^{\frac{1}{2}\phi} \sim \frac{1}{\log u - \left(\frac{7}{8} + \frac{g'(T_0)^2}{g(T_0)^2}\right)\log\log u}\,. \tag{2.17}$$

Let us compare this to perturbative Yang-Mills, where the coupling, up to two loop order is,

$$ {g_{YM}}^2 \sim \frac{1}{\log u - \frac{b_2}{2b_1^2}\log\log u} \tag{2.18} $$

and where b_1 and b_2 are the one and two loop contributions to the β-function.[1] For an SU(N) gauge theory with six adjoint scalars, the ratio is $b_2/2b_1^2 = 3/16$. Hence we find that the sign of $b_2/2b_1^2$ coming from dual gravity is model independent, and that the minimum value of the ratio, and the one that comes closest to the perturbative result, is 7/8 when $g'(T_0) = 0$.

3. Connecting to the infrared

While it is satisfying that a reasonable *UV* solution exists, it is not immediately clear what the *IR* behavior is like. Numerical simulation is very difficult with four coupled nonlinear differential equations, so while useful, it does not immediately lead one to the answer.

To better explore the situation let us consider a greatly simplified model where $f'(T_0) = g(T_0) = g'(T_0) = 0$. In this case the tachyon is constant and the other three fields decouple from each other. Obviously the solution for the dilaton is $\phi = \alpha_0\rho + \phi_0$. The other equations of motion reduce to

$$ \ddot{\xi} + \frac{2Q^2}{f(T_0)} e^{-2\xi} = 0 \ddot{\eta} + 8e^{-4\xi} = 0\,. \tag{3.1} $$

The $AdS_5 \times S_5$ solution is $\xi = \log(\sqrt{2/f(T_0)}Q\rho$ and $\eta = \frac{1}{2}\log(4\rho)$. However, there can be other solutions to (3.1). Consider the case where the solution asymptotically approaches the $AdS_5 \times S_5$ solution in the limit $\rho \to 0$. Each second order differential equation should have two integration constants. Two of these are fixed by choosing the singularity at $\rho = 0$. These generalized solutions are given by

$$ \xi = \log\left(\sqrt{\frac{2}{f(T_0)}}\,Q\rho\right) + \sum_{n=1} a_n\rho^{2n} \eta = \frac{1}{2}\log(4\rho) + \sum_{n=1} b_n\rho^{2n}\,, \tag{3.2} $$

where a_1 and b_1 are free parameters and the higher a_n and b_n are determined by recursion relations.

We can also consider corrections to the *IR* ($\rho \to \infty$) $AdS_5 \times S_5$ solution. In this case the solutions have the form

$$ \xi = \log\left(\sqrt{\frac{2}{f(T_0)}}Q\rho\right) + \sum_{n=1} c_n\rho^{-n} \eta = \frac{1}{2}\log(4\rho) + \sum_{n=1} d_n\rho^{-n}\,, \tag{3.3} $$

[1] A more careful calculation of the effective coupling from the Wilson loop shows that there are no additional $\log\log u$ terms. We use the convention that b_1 is negative and b_2 is positive.

where c_1 and d_1 are free parameters and the higher c_n and d_n are computed via recursion relations. It is straightforward to show that a *UV* solution with generic values for a_1 and b_1 does not connect to the *IR* solutions in (3.3) for any value of c_1 and d_1. To see this, note that ξ and η both satisfy an invariance equation,

$$\frac{1}{2}\dot{\xi}^2 - \frac{Q^2}{f(T_0)}e^{-2\xi} = C_1\,, \qquad -5\dot{\eta}^2 + 20e^{-4\eta} = C_2\,. \tag{3.4}$$

A quick calculation shows that for the *UV* solutions, $C_1 = 2a_1$ and $C_2 = -10b_1$. However, the *IR* solution has $C_1 = C_2 = 0$ for any c_1 and d_1. Hence, unless $a_1 = b_1 = 0$, the *UV* solution does not connect to this *IR* solution. To see what solution it does connect to, note that if $\xi \gg 1$, then the nonlinear term in the equation of motion is small, likewise for the equation for η. Hence for large ξ, the leading order behavior for ξ and η is $\xi = \alpha_1\rho + \xi_0 + \mathrm{O}(e^{-2\alpha_1\rho})$ and $\eta = \alpha_2\rho + \eta_0 + \mathrm{O}(e^{-4\alpha_2\rho})$, with $\alpha_1, \alpha_2 > 0$. Note that these solutions are quite generic since they involve 4 integration constants. The constants of the motion are $C_1 = \alpha_1^2/2$ and $C_2 = -5\alpha_2^2$. Hence we see that the *UV* solutions connect to the *IR* solutions with $\alpha_1 = 2\sqrt{a_1}$ and $\alpha_2 = \sqrt{2b_1}$. If we include the α_0 term from ϕ, then the constraint condition in (2.3) implies

$$\frac{1}{2}\alpha_0^2 + \frac{1}{2}\alpha_1^2 - 5\alpha_2^2 = 0\,. \tag{3.5}$$

From the *UV* point of view, after fixing the coupling and the singularities at $\rho = 0$, we are left with three integration constants. The constraint reduces this to two. Adjusting these integration constants adjusts the values of α_i, $i = 0, 1, 2$, which also satisfy a constraint relation. Notice that one possible transformation is the conformal rescaling, where

$$\xi(\rho) \to \xi(t\rho) - \log(t)\,, \qquad \eta(\rho) \to \eta(t\rho) - \frac{1}{2}\log(t)\,, \qquad \phi(\rho) \to \phi(t\rho)\,. \tag{3.6}$$

Obviously, under this transformation the α_i rescale to $\alpha_i \to t\alpha_i$, and so the α_i can be set arbitrarily close to zero, the limiting result being the usual *IR* fixed point of $AdS_5 \times S_5$. Aside from this conformal transformation, we are left with one independent transformation to the integration constants.

Let us turn to the more difficult case where $f'(T_0) = g'(T_0) = 0$, but $g(T_0) \neq 0$. From the last line of (2.6) we see that T is constant, hence the problem is reduced to 3 coupled nonlinear differential equations. As was argued in the last section [26, 33], there is a *UV* solution whose leading behavior is given by (2.16). Adjusting ξ, η and ϕ to all have singularities at $\rho = 0$ takes care of three integration constants. Another integration constant is varied under the conformal rescalings in (3.6). This leaves two integration constants. These are of a similar type as the previous example, although they are entangled due to the coupling of the equations. The leading correction for ξ

is $5a\rho^2$ while the leading correction to η is $a\rho^2$. These also generate corrections to ϕ. There is another integration constant appearing at order ρ^2 which can be adjusted to satisfy the constraint in (2.3), leaving only one adjustable integration constant.

Notice that since the *UV* effective coupling is $g^2_{eff} \sim 1/(\log u)$, these subleading terms look like instanton contributions. In the perturbative regime, we should expect instanton contributions to be powers of $u^{-8N/3} = \rho^{2N/3}$. Hence in the large N limit, no such terms should appear.[2] Since all fields lie in the adjoint representation, it is possible that there are fractional instantons. However, these usually require fermion zero modes which are conspicuously absent here.

Another source for u^{-1} corrections are scalar masses. These could be bare masses or masses generated by quantum corrections. The ability to adjust the integration constants on the type 0 gravity side could then correspond to the freedom to adjust the bare masses on the field theory side.[3] The powers of u could also just be an artifact of the strong coupling expansion. In any event, since this theory is not conformally invariant, there does not seem to be any *a priori* reason why they should be set to zero. Hence, we will assume that these coefficients are fixed, but generic.

Let us now try to match to *IR* solutions. One such solution was discussed in [26]. Consider the ansatz where $e^{-4\eta}$ is small compared to the other terms in V. This corresponds to a small curvature for the 5 sphere. Dropping this term, one can now find an exact solution to the equations of motion that satisfies the constraint. The solution is

$$e^{\phi} = C_0^2 \rho^{5/9}\,, \qquad e^{\eta} = C_2 \rho^{5/9}\,, \qquad e^{\xi} = \frac{3Q}{\sqrt{2f(T_0)}}\rho\,, \tag{3.7}$$

with the relation

$$10(2f(T_0))^{1/4} C_2^5 - 9g(T_0)\sqrt{3Q}\,C_0 = 0\,. \tag{3.8}$$

Comparing all terms in V, one has $e^{\frac{1}{2}\phi+\frac{1}{2}\xi-5\eta} \sim \rho^{-2}$, $e^{-2\xi} \sim \rho^{-2}$, but $e^{-4\eta} \sim \rho^{-20/9}$. Hence this solution is valid for large ρ. From (3.7), the coupling blows up as $\rho \to \infty$ and after substituting $\rho = 1/u^4$ the metric is

$$ds^2 = \frac{10}{9g(T_0)}\left(16\frac{du^2}{u^2} + \frac{C_2^5\sqrt{2f(T_0)}}{3Q}u^{8/9}dx_{||}^2 + C_2^5 u^{-8/9} d\Omega_5^2\right). \tag{3.9}$$

Here C_2 remains as a leftover integration constant. As in the *UV*, the curvature in the *IR* is small if $g(T_0) \ll 1$. However, there is no reason to expect $g(T_0)$ not to be of order unity. Hence, as in the *UV*, the string corrections are of the same order as the classical contribution.

[2] Unless of course $N = 3$!

[3] I thank I. Klebanov for a comment suggesting this.

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While e^{ϕ} blows up for this particular *IR* solution, one can easily see that the effective coupling found from the heavy quark potential reaches a finite value. Defining a new variable v such that

$$v = \frac{1}{9}\left(\frac{\sqrt{2f(T_0)}}{3Q}\right)^{1/2} u^{4/9}, \tag{3.10}$$

the metric in (3.9) is then

$$ds^2 = \frac{90}{g(T_0)}\left(\frac{dv^2}{v^2} + v^2 dx_{||}^2 + \frac{\left(\frac{C_2}{3}\right)^{10}\sqrt{2f(T_0)}}{Q^2 v^2} d\Omega_5^2\right). \tag{3.11}$$

From this metric, we see that $R^2 = \frac{90}{g(T_0)}$, and so the heavy quark potential is [26]

$$V \approx -\frac{720\,\pi^3}{\Gamma(\frac{1}{4})^4 g(T_0) L} \qquad L \gg L_0\,. \tag{3.12}$$

Hence, we see that this particular solution is a conformal fixed point in the *IR*.

A possible cause of this behavior is that the SU(N) gauge group has been Higgsed to $U(1)^{n-1}$ by the adjoint scalars. Naively, one expects a repulsive force between the branes because of the extra R-R field. In the large N limit, the branes could be pushed apart, but still maintain the $SO(6)$ symmetry. The effect would be to have the branes smeared out over some region with spherical symmetry and finite size. In the UV, well above the Higgs scale, the gauge group is unbroken and the coupling runs. In the infrared, the Wilson loop will probe down into this smeared region and see the effect of the broken gauge symmetry, hence the quark potential behaves coulombically.

However, this ignores the role of the tachyon, which one might expect to lead to attraction between the branes. From the perturbative side this seems reasonable, since quantum fluctuations would give masses to the scalar fields, removing the flat directions away from the unbroken gauge theory. There is a Coleman-Weinberg potential that is unstable [38], but this requires fine tuning to get rid of the scalar mass.

Connecting this *IR* solution to the desired UV solution is problematic. Just as in the uncoupled case, the generic *IR* behavior for $\rho \to \infty$ has the form

$$\begin{aligned} \phi &\approx \alpha_0 \rho + \phi_0 \\ \xi &\approx \alpha_1 \rho + \xi_0 \\ \eta &\approx \alpha_2 \rho + \eta_0\,. \end{aligned} \tag{3.13}$$

The constraints on α_i are

$$\alpha_i \geq 0\,, \qquad \frac{1}{2}\alpha_0^2 + \frac{1}{2}\alpha_1^2 - 5\alpha_2^2 = 0\,, \tag{3.14}$$

where the latter constraint also insures that $5\alpha_1 > \frac{1}{2}\alpha_1 + \frac{1}{2}\alpha_2$, thus insuring that all terms in the potential V have an exponential falloff. The UV solution is connected to the IR solution in (3.7) by varying integration constants such that the α_i in (3.13) are set to zero. From the IR side of things, we see that the α_i can all be set to zero by an infinite rescaling. But we do not want to do this since the rescaling will take the UV coupling to zero at finite ρ. Hence we only have one free parameter to work with. Thus we see that it is not even guaranteed that that there exists an a_1 and b_1 that will connect the UV solution to the IR conformal point. In fact, without some symmetry, it seems highly unlikely. Even if it is possible to set the α_i to zero, it is at best a horrendous fine tuning problem.

Let us thus accept that the UV solution attaches to an IR solution with the asymptotic behavior in (3.13), with nonzero α_i. In section 5 we will investigate the consequences of this. Before doing this let us round out this section by discussing the most general case where $g'(T_0) \neq 0$. In this case T will flow as ρ is varied and thus we have 4 coupled nonlinear differential equations. If $g'(T_0) < 0$ then T flows toward zero as ρ is increased. One possible IR solution was discussed in [33] where ξ and η increase logarithmically, ϕ increases as a log of a log, and T relaxes to zero. The nice thing about this scenario is that the behavior of $g(T)$ and $f(T)$ are known as $T \to 0$. Unlike the previous case, there appear to be enough integration constants in the UV to match solutions. The generic solution is still of the form (3.13) with the additional equation

$$T \approx \alpha_3 \rho + T_0 \,. \tag{3.15}$$

The constraint equation restricts the α_i as

$$\frac{1}{2}\alpha_0^2 + \frac{1}{2}\alpha_1^2 + \frac{1}{4}\alpha_3^2 - 5\alpha_2^2 = 0 \,. \tag{3.16}$$

The leading correction to T is $\sqrt{\rho}(T_+\rho^{\beta_+} + T_-\rho^{\beta_-})$, where

$$\beta_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{4f''(T_0)}{f(T_0)}}\right) \,. \tag{3.17}$$

If $f''(T_0) > f(T_0)/4$, which is the case for the function in (2.4), then we can write down an oscillatory solution with an amplitude and a phase shift. Hence, naively anyway, we appear to have enough integration constants to match solutions. But matching to this particular IR solution still requires a tremendous amount of fine tuning.

If $f''(T_0) \leq f(T_0)/4$ then $\beta_\pm$ would both be positive real and there would still be two integration constants to adjust. This behavior would also fix the problem of instabilities due to the negative tachyon mass near the UV fixed point. At this point, the space is locally AdS_5 and so the condition for stability is that the tachyon mass satisfy $m^2 \geq -4$ [39, 40, 3]. This is precisely the condition that $f''(T_0) \leq f(T_0)/4$. If this condition were satisfied then at the UV fixed point the tachyon would couple to a relevant operator.

More generic is the case where T relaxes to zero or some other constant T_{IR} while the other variables have the asymptotic behavior in (3.13). In this case, $T - T_{IR}$ falls off exponentially as $\rho \to \infty$. We can see from the equations of motion for T that the second derivative starts out positive, but becomes negative at some point before $T = 0$. Hence, it should be possible to find a solution where T starts upward but then relaxes to some finite point, while all the other fields grow linearly in ρ.

Finally, let us consider $g'(T_0) > 0$. Now T flows away from $T = 0$ and there appear to be no possible IR solutions except for those of (3.13) with $\alpha_i > 0$ and (3.15) with $\alpha_3 < 0$.

4. Possible validity for a class of infrared solutions

In order to trust the solutions with the asymptotic behavior in (3.13) and (3.15), it is necessary to check that the higher α' corrections are small. At first glance, this would appear to be a miserable failure. The curvature in the Einstein metric $ds_E^2 = e^{-\phi/2} ds^2$, where ds^2 is the metric in (2.1), approaches

$$R \approx \left(\frac{1}{2}{\alpha_1}^2 - 5{\alpha_2}^2\right) e^{(5\alpha_2 - \alpha_1/2)\rho} . \tag{4.1}$$

This blows up as $\rho \to \infty$ because of the constraint in (3.16).

The leading string corrections are of the form $\alpha'^3 e^{-3\phi/2} W$, where W is a combination of four contracted Riemann tensors. Hence the naive dependence for the α'^3 terms using the solutions in (3.13) is

$$e^{-3\phi/2} W \sim e^{(20\alpha_2 - 2\alpha_1 - 3\alpha_0/2)\rho} . \tag{4.2}$$

Using the conditions in (3.16), one immediately sees that this term grows faster than R, and hence the classical approximation breaks down.

However, W has a field redefinition ambiguity and can be written strictly in terms of the Weyl tensor $C_{\mu\nu\lambda\delta}$. For the Einstein metric, the nonzero components of the Weyl tensor have the form

$$C_{\mu\nu\mu\nu} = g_{\mu\mu} g_{\nu\nu} (A_1 \mathcal{F} + A_2 \mathcal{G}) e^{5\eta - \xi/2} , \tag{4.3}$$

where A_1 and A_2 are constants (different components have different constants), and $\mathcal{F}$ and $\mathcal{G}$ are defined as

$$\mathcal{F} = 2(\dot{\xi} - \dot{\eta})\dot{\eta} + \ddot{\xi} - \ddot{\eta} , \qquad \mathcal{G} = \dot{\xi}^2 - \dot{\eta}^2 + \ddot{\xi} - \ddot{\eta} - 4e^{-4\eta} . \tag{4.4}$$

It is clear that for a general solution in 3.13, both $\mathcal{F}$ and $\mathcal{G}$ are of order unity, and so $e^{-3/2\phi} W$ blows up as in (4.2). However, if $\alpha_2 = \alpha_1$, then both terms are suppressed. We actually should have anticipated this, since now the Einstein metric approaches that of the product space $R_+ \times M_4 \times S_5$, up to a conformal transformation. Recall that the metric for the 3(4) dimensional Witten model approaches the metric for the product space $R_5 \times S_5$ ($R_6 \times S_4$) near the black hole horizon.

However, simply setting $\alpha_2 = \alpha_1$ is not sufficient to insure that $e^{-3\phi/2}W$ blows up slower than R. From the equations of motion in (2.6), we see that ξ has corrections of order $e^{-2\alpha_1\rho}$ and η has corrections of order $e^{(\alpha_0+\alpha_1-10\alpha_2)\rho/2}$. Letting $\alpha_2 = \alpha_1$, we find in general that $\mathcal{F} \sim e^{-2\alpha_1\rho}$ and $\mathcal{G} \sim e^{-2\alpha_1\rho}$, and thus

$$e^{-3\phi/2}W \sim e^{(10\alpha_1 - 3\alpha_0/2)\rho}\,. \tag{4.5}$$

While the behavior has been softened, it is still not enough to prevent the α'^3 correction from dominating R, since the maximum value for α_0 consistent with (3.16) is $\alpha_0 = 3\alpha_1$.

However, there are solutions where W grows even slower than (4.5). If T approaches a constant T_{IR} as $\rho \to \infty$, then $\alpha_3 = 0$ and the asymptotic solution has the relations

$$\alpha_0 = 3\alpha_1\,, \qquad \alpha_2 = \alpha_1\,. \tag{4.6}$$

For this case, ξ has corrections of order $e^{-2\alpha_1\rho}$ and $e^{-3\alpha_1\rho}$ while η has corrections of order $e^{-3\alpha_1\rho}$. It is a straightforward excercise to check that these corrections cancel in *both* $\mathcal{F}$ and $\mathcal{G}$ and therefore

$$e^{-3\phi/2}W \sim e^{-5\alpha_1\rho/2}\,. \tag{4.7}$$

Hence, not only does the term grow slower than R, it actually falls off exponentially!

We can also check the behavior for other types of terms. One class of terms has the form [33]

$$e^{\phi/2}\left(e^{-\phi/2}C\right)^n \sim e^{-(2n-3)\alpha_1\rho/2}\,, \tag{4.8}$$

falling off even faster than the first string correction. There are also terms of the form

$$e^{-(n-1)\phi/2}(\nabla\phi\nabla\phi)^n \sim e^{-3(n-1)\alpha_1\phi/2}\,, \tag{4.9}$$

which again falls off rapidly. Terms involving derivatives of the tachyon field also have exponential suppression since T approaches a constant value with exponential falloff. The behavior of all these corrections leads us to speculate that there is a background that is an exact solution of the σ model which has the asymptotic behavior defined by (3.13) and (4.6).

There is some fine tuning involved in connecting the desired *UV* solution to the *IR* solutions in (4.6). We need to tune such that $\alpha_1 = \alpha_2$ and $\alpha_3 = 0$. If $g'(T_0) = 0$ then $\alpha_3 = 0$ is automatically satisfied. In either case there are enough available integration constants on the *UV* side which can be adjusted so that (4.6) is satisfied. Hence, we believe that there is a *UV* solution that connects to the *IR* solutions in (3.13) and (4.6).

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5. Confinement in the infrared

In this section, we argue that the *IR* solutions of (3.13) and (3.15) lead to a linear quark potential, magnetic screening and a discrete spectrum. The linear quark potential requires the additional condition $\alpha_1 \geq \alpha_0$. Happily, the solution in (4.6) satisfies this condition. It is possible that other solutions besides (4.6) are valid, perhaps there is enough ambiguity in the effective action to make their α' corrections small as well. To allow for this, we will discuss the general case in this section.

5.1 The quark potential

The quark potential is computed using the Wilson loop calculation of [36, 37]. The Nambu-Goto action for a string in a curved background is given by

$$S = \frac{1}{2\pi} \int d\sigma \delta\tau \sqrt{\det(G_{MN} \partial_\alpha X^M \partial_\beta X^N)} \,. \tag{5.1}$$

The quark potential is computed from the rectangular Wilson loop, with two sides along the time direction and the other two along one of the spatial directions. Using the metric in (2.1), we find that the energy for the quark-antiquark pair is

$$E = \frac{1}{2\pi\sqrt{Q}} \int dx \sqrt{e^{\phi - 5\eta} \left(\frac{\partial\rho}{\partial x}\right)^2 + e^{\phi-\xi}} \,. \tag{5.2}$$

Changing variables such that $du = -e^{(\phi-5\eta)/2} d\rho$, we have

$$E = \frac{1}{2\pi\sqrt{Q}} \int dx \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + f(u)/Q} \,, \tag{5.3}$$

where $f(u)$ has the asymptotic behavior

$$\begin{aligned} f(u) &\sim \frac{u^4}{(\log u)^2}\,, \qquad u \gg 1 \\ f(u) &\sim u^\gamma\,, \qquad \gamma = \frac{2(\alpha_1 - \alpha_0)}{5\alpha_2 - \alpha_0}\,, \qquad u \ll 1\,. \end{aligned} \tag{5.4}$$

Assuming that all $\alpha_i \geq 0$, the constraint equation (3.16) implies that the exponent for the small u behavior satisfies $\gamma < 2$. If $\alpha_1 > \alpha_0$, then $0 < \gamma < 2$, and thus for large enough x, the minimum energy configuration consists of a string starting at $u = \infty$ and going straight down to the origin at $u = 0$, traveling a distance x along $u = 0$, and then going back out to $u = \infty$. Since $f(u) = 0$ at $u = 0$, there is no cost in energy to separate the quarks further, thus this would correspond to electric screening [41, 42].

If instead $\alpha_1 = \alpha_0$, then u^γ is no longer zero at the origin. In this case, for large enough x the string will approach the origin and the quark potential becomes linear. If $\alpha_1 < a_0$, then u^γ blows up at the origin. In this case there will be some nonzero u_0 such that $f(u_0)$ is a minimum. For large enough x the string will approach this minimum value. Since $f(u)$ is clearly positive for nonzero u, this too will lead to a linear quark potential. Assuming that the α_i are all of order 1, then the string tension will be of order $1/Q$ in the units used here.

If $\alpha_1 < \alpha_0$, then the coupling seen by the string is in some sense bounded in the infrared. For large quark separation, the minimum energy configuration has $u > u_0$, thus the coupling approaches $e^{\phi_0+\alpha_1 u_0}$. This is not to say that the behavior of the metric and coupling for $u < u_0$ will not affect other physical results, as we will see in the following subsections.

5.2 Magnetic screening

If there is confinement, then there should be a corresponding screening of magnetic charge. The computation is along the lines of [17]. In particular, we wish to compute the potential between a monopole anti-monopole pair. This is accomplished by calculating the rectangular Wilson loop for a D string. In type 0 theory, since there are two types of R-R Kalb-Ramond fields, there are also two types of D strings. However, only one type of D string can end on the electric D3 brane. It is this string that describes the Wilson loop for a monopole.

The calculation is almost identical to the Wilson loop calculation for the fundamental string. The only difference is that the integrand in (5.1) is multiplied by a factor of $e^{-\phi}$. Hence, the potential is given by

$$E = \frac{1}{2\pi}\int dx \sqrt{e^{-\phi-5\eta}\left(\frac{\partial\rho}{\partial x}\right)^2 + e^{-\phi-\xi}}\,. \tag{5.5}$$

This time changing variables such that $du = e^{(-\phi-5\eta)/2}d\rho$, we have

$$E = \frac{1}{2\pi}\int dx \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + f(u)/Q}\,, \tag{5.6}$$

where now $f(u)$ has the asymptotic behavior

$$\begin{aligned} f(u) &\sim u^4(\log u)^2\,, \qquad u \gg 1 \\ f(u) &\sim u^\gamma \qquad \gamma = \frac{2(\alpha_1+\alpha_0)}{5\alpha_2+\alpha_0}\,, \qquad u \ll 1\,. \end{aligned} \tag{5.7}$$

Since all $\alpha_i \geq 0$ and $5\alpha_2 > a_1$, then $0 < \gamma < 2$. Hence, the monopole and anti-monopole will screen at large distances

5.3 WKB estimates for the glueball spectrum

The correlator $\langle \mathrm{Tr}(F^2(x))\mathrm{Tr}(F^2(0))\rangle$ is related to scalar Green's functions in the bulk [2, 3]. A discrete spectrum for the bulk particles would correspond to a discrete spectrum for the scalar glueballs [16]. Since we are dealing with string tensions of order 1, the green's functions of massive string states could be important. More to the point, the tachyon couples to $\mathrm{Tr}(F^2)$ [33, 43]. If the tachyon is a bad tachyon, with plane wave normalizable states in the bulk, then it cannot have a discrete spectrum. Even if it is well behaved with $f''(T_0) < f(T_0)/4$, one might find that the lowest lying state in the spectrum is tachyonic.

The linear equations one derives for the small fluctuations are hopelessly entangled through the tachyon mass term. As a first approximation, one can ignore the cross terms and simply consider the diagonal equations. Let us make the usual ansatz and assume that the 0^{++} glueball spectrum is determined by the spectrum for small fluctuations of the dilaton equation of motion. Consider dilaton solutions of the form $\phi = \psi(\rho)e^{ik\cdot x}$, where $k^2 = -M^2$. Then the dilaton equation of motion reduces to

$$\frac{\partial}{\partial\rho}\sqrt{g}e^{-2\phi}g^{\rho\rho}\frac{\partial}{\partial\rho}\psi + M^2\sqrt{g}e^{-2\phi}g^{xx}\psi + \frac{1}{4}g(T)\sqrt{g}\psi = 0\,, \tag{5.8}$$

where ϕ in (5.8) refers to the background dilaton field. Substituting the metric in (2.1) into (5.8) and using the equation of motion for the background dilaton field, one finds

$$\frac{\partial^2}{\partial\rho^2}\psi + M^2e^{\xi-5\eta}\psi - \frac{1}{2}\phi''\psi = 0\,. \tag{5.9}$$

Defining u as $e^{\xi-5\eta} = Qu^6\sqrt{2/f(T_0)}/32$, the equation of motion becomes

$$\frac{\partial}{\partial u}F(u)\frac{\partial}{\partial u}\psi + M^2QH(u)\psi + G(u)\psi = 0\,, \tag{5.10}$$

where $F(u)$, $H(u)$ and $G(u)$ have the asymptotic behavior

$$\begin{aligned} &F(u) \approx u^5\,, \qquad H(u) \sim u\,, \qquad G(u) \approx \frac{4u^3}{\log u}\,, \qquad u \gg 1 \\ &F(u) \sim u\,, \qquad H(u) \sim u^5 \qquad G(u) \sim u^{7/2}\,, \qquad u \ll 1\,. \end{aligned} \tag{5.11}$$

Note that the substitution for u requires that $5\alpha_2 > \alpha_1$ in order that $u \to 0$ as $\rho \to \infty$. Luckily, this inequality is guaranteed by (3.16).

Using the arguments in [16], one can argue that the spectrum is discrete. In fact, we can say more. Since we know the asymptotic behavior, one can do the WKB calculation described in [26]. The asymptotic behavior is similar to that of the three dimensional Witten model [16], although there will be log corrections because of the UV behavior of $G(u)$. The $G(u)$ term does not affect the IR turning point to next to leading order. Borrowing the results in [26] and inserting the $G(u)$ correction, we find that the WKB 0^{++} glueball spectrum is

$$M^2 = \frac{4C\sqrt{2f(T_0)}}{Q}\, m\left(m+1-\frac{2}{\log[Cm(m+1)]}\right) + \mathrm{O}\left(\frac{m}{\log^2 m}\right), \qquad m \geq 1\,, \tag{5.12}$$

where C is a constant of order unity. Corrections to (5.12) coming from the inclusion of cross terms are of order $m/\log^2 m$. The log factors in (5.12) have the effect of dragging down the glueball masses, which is clearly caused by the running of the coupling. This is especially true for the lightest state. If $C = 1$ and one assumed that (5.12) were exact, then one would find that the lightest state is actually tachyonic. At this point we are unable to prove that a tachyonic state does not appear. We can say that if a tachyonic state were to exist for the dilaton equation in (5.8), then a minimum requirement is that there is some region where $\rho^2\phi'' < -1/2$. One can see that this is never true if we only consider the leading order term in (2.16) for ϕ. Numerical analysis seems to show that it is not true if the higher order terms are also included. Once we are safely in the infrared, then ϕ'' will be exponentially suppressed and so $\rho^2\phi'' > -1/2$ in this region as well. However, inclusion of the cross terms might drop the masses enough to leave a tachyonic glueball state. Hopefully this does not occur.

Since $Q \sim N$, we see that the glueball masses are of the same order as the square root of the string tension. This compares favorably to the 4 dimensional Witten model, where the masses are a factor of $N^{1/2}$ smaller [44]. We can also compare the ratio of the first excited glueball to the lowest mass state. Since our knowledge of C is limited, we can only put a lower bound on the mass ratios, which is reached when $C \to \infty$. Plugging in numbers, we find that this ratio satisfies $m^*_{++}/m_{++} > \sqrt{3} \approx 1.73$. The WKB approximation for this ratio in the 4 dimensional Witten model is $\sqrt{8/3} \approx 1.63$ [26]. The SU(3) lattice result is $1.77 \pm .14$ [19].

We can also make WKB approximations for the glueball states with nontrivial $SO(6)$ quantum numbers [45]. If we write the dilaton field as $\phi = \psi(\rho)e^{ik\cdot x}Y_l(\Omega_5)$, then the dilaton equation of motion reduces to

$$\frac{\partial^2}{\partial\rho^2}\psi + M^2 e^{\xi-5\eta}\psi - l(l+4)e^{-4\eta}\psi - \frac{1}{2}\phi''\psi = 0\,. \tag{5.13}$$

If $\alpha_1 \geq \alpha_2$, which is the case for (4.6), then the M^2 term will dominate the $\ell(\ell+4)$ term as $u \to 0$. In this case, the WKB calculation is a straightforward generalization of the $l = 0$ result and is

$$M^2 = \frac{4C\sqrt{2f(T_0)}}{Q}\, m\left(m+1+l-\frac{2}{\log[Cm(m+1)]}\right) + \mathrm{O}\left(\frac{m}{\log^2 m}\right), \qquad m \geq 1\,. \tag{5.14}$$

If $\alpha_1 < \alpha_2$, then the asymptotic behavior changes near $u = 0$, but this effect only leads to corrections of order 1 in m.

We complete this subsection by computing the WKB spectrum for the 0^{-+} glueballs. In the Witten model, this was done by studying the wave function for the 5 dimensional dual of the NS-NS two form field, which is a vector [20, 23]. The component along the Euclidean time direction is a scalar in the four dimensional theory and is odd under parity. In the case that we are studying, there is no euclidean circle, so we need to consider instead the wave function for a scalar field. The scalar field is the IIB axion which is an R-R scalar. Actually, there are two scalars, but only one couples to $\mathrm{Tr}(F\widetilde{F})$ on the electric D3 branes. The other scalar couples to $\mathrm{Tr}(F\widetilde{F})$ on the magnetic D3 branes.

Since the axion comes from the R-R sector, it does not couple to the dilaton.[4] Hence, the wave function has the form

$$e^{-2\phi}\frac{\partial}{\partial\rho}e^{2\phi}\frac{\partial}{\partial\rho}a(\rho)+M^2e^{\xi-5\eta}=0\,. \tag{5.15}$$

Defining $a(\rho)=e^{-\phi}\psi(\rho)$, (5.15) can be rewritten as

$$\frac{\partial^2}{\partial\rho^2}\psi+M^2e^{\xi-5\eta}\psi\ -\left(\phi''+\phi'^2\right)\psi=0\,. \tag{5.16}$$

As was the case for the dilaton, the ϕ terms do affect the next to leading order contribution of the UV turning point. The ϕ'^2 term also changes the next to leading order contribution of the IR turning point. Using the analysis in [26], one finds that the WKB mass expression for the 0^{-+} states are

$$M^2=\frac{4C\sqrt{2f(T_0)}}{Q}m\left(m+1+\frac{2\alpha_0}{5\alpha_2-\alpha_1}-\right.$$
$$\left.-\frac{4}{\log[Cm(m+1)]}\right)+\mathrm{O}\left(\frac{m}{\log^2 m}\right),\qquad m\geq 1 \tag{5.17}$$

where C is the same constant as in (5.12). Unlike the 0^{++} states, but like the 0^{-+} states in the Russo generalization of the Witten model [24, 25, 28], there is model dependence in these glueball masses. Using the solutions in (4.6) we find as a lower bound for the mass ratio $m_{-+}/m_{++}>\sqrt{7}/2\approx 1.32$. The most recent lattice results have $m_{-+}/m_{++}=1.78\pm .24$ for SU(2) ([19]) and $m_{-+}/m_{++}=1.50\pm .04$ for SU(3) [46]

6. Discussion

One of the main differences between the results found here and results found for the Witten model is that the type 0 model has a QCD scale set by dimensional transmutation. Hence, the glueball mass is directly related to the string scale. We have also found differences for the mass ratios. The principle reasons for these

[4]The axion does couple to the tachyon [29], but this does not affect the WKB approximation to next to leading order.

differences is that the type 0 model is four, not five dimensional in the UV, and because the type 0 model has a running coupling constant. The N dependence in the mass to string scale ratio is also affected by the differences in dimension. The Witten model has a factor of $N^{-1/2}$, while the type 0 model has a factor of $N^{-1/4}$

The existence of the confining solutions does not contradict other results for IR fixed points. The fine tuning involved to reach such solutions could correspond to tuning mass parameters on the field theory side, so that for instance, the six scalars are massless after including their quantum corrections. In this case, the field theory could have the IR fixed point described in [33]. The confining solutions, which do not need to be fine tuned, have massive scalars, and so are in the same universality class as pure SU(N).[5]

We have seen that while confinement in the infrared is generic, there exists solutions where many of the string corrections vanish exponentially. One such solution has a constant tachyon expectation value. In some sense, this is closer to the spirit of Polyakov's noncritical string for QCD, where the nonzero tachyon expectation value, "peacefully condensing in the bulk", plays the role of a nonzero central charge [32, 47]. On the other hand, the five form flux through S_5 clearly plays an important part here; without it we would not have the log scaling in the UV.

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